Study and use of Ledger's conditions

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Ledger's conditions are an infinite series of curvature conditions denoted by L_q, $q \geq 2$. It was proved by L. Vanhecke that the "odd" Ledger conditions are already consequences of the "even" ones. Anyway, these inductively defined conditions soon become very complicated. Thus, the explicit form of L_q is known only for small values of q ($q = 3, 5, 7$). Moreover, the difficulty of cheking if they are satisfied or not on a given manifold increases with the dimension of the manifold.

On the other hand these conditions are so important because the property of being a D'Atri space (i.e., a space with volume-preserving symmetries) is equivalent to the odd Ledger conditions. In addition, a Riemannian manifold (M, g) satisfying the first odd Ledger condition is said to be of type $\mathcal{A}$.

Now, we have found a way to study the Ledger conditions on the six and twelve dimensional Wallach's flag manifolds. Moreover, we have used it to determine when they are D'Atri spaces.

In addition, using it between others tools, we present the complete local classification of all 4-dimensional homogeneous spaces of type $\mathcal{A}$ in a simple and explicit form and, as a consequence, we prove correctly that all 4-dimensional homogeneous D'Atri spaces are locally naturally reductive.