Enumerative geometry and string theory

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Enumerative geometry is an area of mathematics which dates back to the nineteenth century and is concerned with the counting of solutions to geometric questions. The aim of this lecture course is to show how ideas from string theory and quantum field theory can be successfully applied to the concrete problem of counting the number of rational curves on certain algebraic varieties. For this some basic notions and techniques from string theory and its formulation as a topological quantum field theory will be introduced. As a result, formulas enumerating rational curves can be derived from the properties of certain generating functions associated to the topological field theory. The lecture series ends with a brief outline of some current research directions which explore the mirror symmetry phenomenon of Calabi-Yau varieties.

Lecture 1. Counting rational curves

Lecture 2. Elements of classical and quantum mechanics

Lecture 3. Supersymmetry and localization

Lecture 4. String theory and topological field theory

Lecture 5. Quantum cohomology and enumerative predictions

Suggested references for preparation and further reading:

D. Cox and S. Katz, *Mirror Symmetry and Algebraic Geometry*, Mathematical Surveys and Monographs, Volume 68, AMS, 1999.

K. Hori, S. Katz, A. Klemm, R. Pandharipande, R. Thomas, C. Vafa, R. Vakil and E. Zaslow, *Mirror Symmetry*, Clay Mathematical Monographs, AMS, Providence, RI, 2003.

M. Kontsevich and Y. Manin, *Gromov-Witten classes, quantum cohomology and enumerative geometry*, Comm. Math. Phys. **164**, 525-562 (1994).

D. Morrison, *Mathematical aspects of mirror symmetry,* in Complex Algebraic Geometry, (*Park City, UT, 1993*) 365-327, IAS/Park City Math. Ser. 3, AMS, 1997.

C. Voisin, *Symétrie Miroir*, Panoramas et Synthèses **2**, Soc. Math. France, Paris, 1996.