Announcement of a course on

Q-curvature

Andreas Juhl

As part of the SFB-project **A6** I will give a course on new developments concerning Q-curvature.

For any (pseudo) Riemannian manifold (M, g) of even dimension n there is a remarkable quantity $Q_n(g)$ called Branson's Q-curvature. It has been introduced about 15 years ago by T. Branson in connection with variational formulas for zeta-regularized determinants of elliptic differential operators. The key property of that curvature invariant is the following: although Q_n involves n derivatives of the metric it only depends linearly on $\varphi \in C^\infty(M)$ under conformal changes $e^{2\varphi}g$. Moreover, the linear dependence is given by a conformally covariant linear differential operator of order n with a power of the Laplacian as leading part. The lower order correction terms depend on the Ricci curvature. It is a challenge to study the structure of these operators and the related Q-curvature.

Q-curvature generalizes scalar curvature. It gives rise to natural higher-dimensional prescription problems (a version of the Yamabe type problem) and geometric flows (generalizing the Yamabe flow) which are intesively studied at the moment. Geometric applications beyond dimension 4 are generally obstructed however by the largely unknown structure of Q-curvature in higher dimension.

In the course I will describe recent progress in understanding the nature of Q-curvature. More precisely, we will describe how it appears in a framework of conformal invariants in the geometry of submanifolds. We will follow the philosopical principle that

Q-curvature is generated by iterated curved Zuckerman translation

of the pull-back map induced by the embedding. This perspective opens the way to many new developments.

The course is an introduction to that idea, its ramifications and consequences. Along the way we will touch upon a variety of topics as: GJMS-operators, Zuckerman translation of homomorphisms of Verma modules (holonomic and semi-holonomic), Cartan's conformal connection, elements of conformal tractor calculus, Einstein metrics, holographic renormalization. We will formulate a series of research problems.

Time: Mondays, 9.15 - 11.00

Place: Rudower Chaussee 25, House 1, Room 012

Start: 23.10.2006

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