Inverse problem for the perturbed harmonic oscillator on the half-line.

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Abstract

We consider the perturbed harmonic oscillator $T_D\psi = -\psi'' + x^2\psi + q(x)\psi$ in $L^2(\mathbb{R}_+)$ with the Dirichlet boundary condition $\psi(0) = 0$, where $q \in \mathbb{H}_+ = \{q', xq \in L^2(\mathbb{R}_+)\}$ is a real-valued potential. We prove that the mapping

$$q \mapsto \text{spectral data} = \{\text{eigenvalues of } T_D\} \oplus \{\text{norming constants}\}$$

is one-to-one and onto. The complete characterization of the set of spectral data which corresponds to $q \in \mathbb{H}_+$ is given. Note that the isospectral sets are described in terms of some weighted ℓ^2 space in contrast to the peturbed oscillator on the whole real line. This result was published in Annales Henri Poincare, 2007, jointly with Evgeny Korotyaev.