Classical and quantum isoperimetric problems solved by inequalities for means of chords

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The talk is motivated by isoperimetric problems arising in quantum mechanics as well as in classical electrostatics. Specifically, we consider the Schrödinger operator in $L^2(\mathbb{R}^2)$ with an attractive interaction supported on a closed curve Γ, formally given by $-\Delta - \alpha\delta(x-\Gamma)$ and ask which curve of a given length maximizes the ground state energy. The second problem concerns a loop-shaped thread Γ in $\mathbb{R}^3$, homogeneously charged but not conducting; we ask about the (renormalized) potential-energy minimizer. Both of them reduce to a purely geometric problem about inequalities for mean values of chords of Γ. We prove an isoperimetric theorem for p-means of chords of curves when $p \leq 2$, which gives an answer to the above questions, and find the critical $p > 2$ for which it ceases to be valid. A discrete analogue of the problem is also considered.

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