On high order conformally covariant powers of the Laplacian

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Abstract. Let (M, g) be a Riemannian manifold. It is well-known that, by modifying the Laplace-Beltrami operator Δ_g by a multiple of the scalar curvature of g, one obtains an operator which is covariant under conformal changes of the metric. The conformal Laplacian - or Yamabe operator - is the first in a sequence of conformally covariant differential operators which are obtained by adding lower order terms to the powers of Δ_g. The second in the sequence was discovered almost 25 years ago and is known as the Paneitz operator. More generally, the existence of the sequence was established by Graham, Jenne, Mason and Sparling in their seminal work from 1992 using the Fefferman-Graham ambient metric. The GJMS-operators P_{2N} led to the notion of Branson's Q-curvature which currently plays a central role in conformal geometry. Despite of their importance not much is known about the structure of the GJMS-operators P_{2N} if the power N exceeds 3.

Although conformal covariance is not preserved under composition, I will show how one can built conformally covariant high order powers of the Laplacian from low order ones. This leads to explicit formula for (at least) the next two cases: $N = 3$ and $N = 4$. Both cases will be presented as special cases of a (conjectural) uniform recursive description of all GJMS-operators.