

Prof. Dr. Dirk Kreimer, IHES, Bures-sur-Yvette

**Talk 1 on October 27, 2010** (Wednesday), 17:15-18:45  
HU, RUD 25, 3.001 (ground floor)

**Title: Chen's Lemma in quantum fields**

---

Abstract: We start by reviewing the role of iterated integrals in the theory of polylogarithms. This also allows us to introduce and define shuffle and quasi-shuffle algebras.

We then introduce Chen's lemma in this context. It is intimately connected to a very simple Hopf algebra structure which gives rise to an abelian character group.

This gives a first example how a variation of scale relates to a certain algebraic structure. This is the starting point of a comparison with the situation for quantum field theory. We will try to push the analogy as far as possible in simple examples.

Mainly, we want to study in glorious technical detail some aspects presented in previous lectures, and in particular try to see how the notion of shuffle algebras and iterated integrals transfer to quantum field theory.

In so doing, we will consider the Hopf algebra of sequences in the letters of a given alphabet, next the Hopf algebra of decorated trees which generalizes the previous one from a linear to a non-linear problem, and then the Hopf algebra of graphs, which is still more general.

Still, understanding sequences, we will argue, allows to understand graphs. Hopefully, we then understand the structure of the renormalization group by analogy with the structure of iterated integrals. In particular, we set it as our goal to see how the leading log expansion in physics generalizes a shuffle identity.

We finish by defining a shuffle algebra structure directly on Feynman graphs. The latter construct came up in joint work with Francis Brown.

---

Literature: 'Chen's iterated integral represents the operator product expansion,' D. Kreimer, Published in Adv.Theor.Math.Phys.3:3,2000, Adv.Theor.Math.Phys.3:627-670,1999.  
e-Print: hep-th/9901099

F. Brown: 'Iterated Integrals in Quantum Field Theory', available via  
<http://math.bu.edu/people/dkreimer/houches/ColumbiaNotes7.pdf>