Inhaltsverzeichnis

1 [A2] Special Geometries and Fermionic Field Equations 2005-2008

1.1 Summary

The research project focuses on Riemannian manifolds carrying geometric structures ('special geometries') which are not defined by covariantly constant objects, for instance contact manifolds, almost Hermitian manifolds, weak G_2-manifolds, and the applications there in string theory. A crucial tool for this investigation are Dirac operators. This not only includes the classical Riemannian Dirac operator, but also generalisations such as Dolbeault's or Kostant's cubic Dirac operator. Most of the structures of concern can indeed be defined by means of spinors, a fact manifesting itself in the field equations that occur in string compactifications.

1.2 Current Knowledge

Starting from the second half of the XX century the French school gravitating around M. Berger began developing the idea that Riemannian manifolds could be broadly classified according to their holonomy group. The term *special (integrable) geometries* has become a customary reference for those which are not of general type. The embryonic idea that parallel spinor fields induce special geometries was already present, yet was not further investigated. At the beginning of the seventies, A. Gray generalised the notion of holonomy, by introducing a systematic way of understanding *non-integrable special Riemannian geometries* that relies on differential forms. The connection between these two approaches became clear in the eighties within the contexts of twistor theory and small eigenvalues of the Dirac operator. The latter was mainly developed by the Berlin school of Th. Friedrich. In the case of Riemannian homogeneous spaces, integrable geometries correspond to symmetric spaces, classified by E. Cartan in the 1920s. The richer class of reductive homogeneous spaces – so far inaccessible to any kind of classification – has been studied intensively since the mid-sixties, and represents a major source for non-integrable geometries.

The interest in both integrable and non-integrable geometries was injected new blood by string theory. On the one hand integrable geometries (Calabi-Yau manifolds, Joyce manifolds etc.) are important for the model proposed by A. Strominger, in which the B-field vanishes so the 'integrability' corresponds to 'vacuum solutions'. If on the other hand one looks for string compactifications with non-zero B-field, seen as deformations of vacuum, then solutions to the Strominger equations can be constructed geometrically using non-integrable geometries.

1.3 Results and their Interpretation

Manifolds are the directions in which our interests are pointing. The first aim of future research is to understand more deeply the theory developed so far, and then implement it by setting up a broader point of view encompassing current knowledge. By now, it is well-established that special geometries - integrable or not - should be treated in a comprehensive way based on the study of metric connections with torsion and their holonomy theory.

Such a far-reaching programme obviously touches many areas, among which it was already shown that the role of Dirac analysis and spin geometry is prominent. The problem of capturing in full those geometries admitting a metric connection with parallel torsion is another such. The issue is definitely worth considering, given that the geometry in question can be reasonably controlled. Besides, the same structures are an endless source of examples, homogeneous or not, as required by the spinorial field equations of string theory, for instance.

In this respect the importance of representation theory (read: differential forms) should not be underestimated, which is why we shall address non-linear representations coming from geometric structures in the near future. The specific intent to start with, is to see under which circumstances low dimensional Lie groups might act more broadly than matrices.