Inhaltsverzeichnis

1 [A3] Singularities in Manifolds with Special Holonomy 2005-2008

1.1 Summary

In essence, the focus of the project A3 is a local one – we study singularities. Motivated by physical phenomena like conifold transitions, one tries to study (or at least to obtain) deformations and, if possible, smoothings of these. In the case of Calabi-Yau varieties arising from Batyrev's construction, we investigated special classes of ambient Fano varieties and established their smoothability in codimension three.

Another point of view is that of Gross and Siebert who try to reduce mirror symmetry to some duality in discrete mathematics. Their approach includes the degeneration of Calabi-Yau varieties to reducible spaces. We considered these non-normal singularities, too, and calculated their infinitesimal deformation spaces. Further, smoothing families $\widetilde{X} \to Y$ of these reducible spaces occur in our theory of T-varieties. Originally created as a generalisation of toric varieties in order to dispose of the right framework to study toric deformations, the total spaces $\widetilde{X}$ arise as certain partial resolutions of the T-varieties under investigation.

The description of T-varities by polyhedral divisors on lower-dimensional varieties helps to understand the configuration of T-orbits. This description was given (in form of an equivalence of categories) in both the local and the global case. Since the versal deformation (base and total space) inherits the group action on the given space, the category of spaces with a fixed group acting on them provides the suitable language for studying deformations, rather than the category of toric varieties itself. In the latter, each variety comes along with its own torus.

The formalism of derived categories provides a particularly elegant framework for investigating mirror symmetry and D-branes. We studied the impact of group actions also in this context. As a result we obtained an equivariant theory of Fourier-Mukai functors, leading to applications in birational geometry.

¿ From a Riemannian viewpoint, Calabi-Yau manifolds varieties are only one example of *special metrics* which appear in the context of String-, M- or F-theory compactifications. We investigated also special metrics arising in the context of Hitchin's variational principle. In particular, we found non-trivial compact examples of so-called $PSU(3)$-metrics. Further, D-branes materialise as calibrated submanifolds which generalise the algebraic geometric notion of a complex subvariety and comprise, among other examples, special Lagrangian submanifolds. In view of generalising Floer homology to so-called G_2-metrics, we studied the deformation behaviour of their calibrated submanifolds under boundary conditions.

Eventually, generalized Albanese varieties have been constructed. They provide a contraction of all cycles not contributing to linear representations of the fundamental group of a projective variety.

1.2 Current Knowledge

1.2.1 Smoothings and Degenerations of CYs

Batyrev's construction of Calabi-Yau spaces out of reflexive polytopes yields at first mostly singular varieties. Afterwards, the decent mirror pairs result from crepant resolutions of both partners. Another method to produce smooth Calabi-Yau varieties out of singular ones is to consider deformations along a flat family. Miles Reid even conjectures that the moduli space of Calabi-Yau spaces is connected after allowing the conifold-transitions mentioned above, i.e., both resolutions/contractions and deformations/degenerations. Moreover, contraction plus smoothing is mirror dual to degeneration plus resolution.

While resolutions do always exist and even crepantness at least can be simulated via motivic integration, the construction of smoothings remains a problem. Since one is eventually interested in the smoothing of 3-dimensional Calabi-Yau varieties which are embedded as complete intersection in higher-dimensional Fano varieties, this leads to the following problem: Given an arbitrary Fano variety, for instance by a reflexive polytope, is it then possible to find a deformation with the general fiber being smooth in codimension three?

In [Nam97], Namikawa has shown that Gorenstein Fano threefolds with at most terminal singularities are smoothable. However, to say something about 3-dimensional Calabi-Yau varieties, one needs to consider Fanos of dimension ≥ 4.

Another direction of research is the investigation of reducible spaces. Their smoothability has first been considered by Friedman in the special case of simple normal crossings, [Fri83]. The striking notion in this context was that of d-semistability. Later on, the setting was generalized in [YN94] and [GS07]. The latter paper is just the last in a row of papers dealing with the idea of reducing mirror symmetry to the discrete Legendre transformation. These papers relate degenerating families of Calabi Yau or Fano varieties to certain topological manifolds with an affine structure (outside codimension 2) and a polyhedral subdivision. In particular, the question of smoothability is dealt with.

However, both [YN94] and [GS07] use the concept of smooth log structures. Hence, it is often not clear if the assumptions of their theorems apply for a given situation. For example, even if the reducible variety comes from a simplicial complex via the Stanley-Reisner construction, the log method does not give any information about smoothability yet. Moreover, it is not yet clear how to see the toric degenerations of Grassmannians in their language.

1.2.2 The category of T-Varieties

In the last 20 years, the investigation of toric varieties grew to an important branch of algebraic geometry. It provides nice classes of examples that are approachable by combinatorics, and so theorems (and conjectures) in algebraic geometry can be translated into a completly different language. This does often give new insight into the original problem.

The strength of toric geometry comes from the action of big tori on the varieties – creating a huge amount of redundancy in the geometry. This means, the variety does almost look like the torus itself, and the actual difference can be measured by combinatorics. Algebraically spoken, the torus action corresponds to a very fine multigrading of the coordinate rings. The homogeneous pieces become 1-dimensional (hence boring) vector spaces, and all information about the rings is completely hidden in the structure of the set containing the degrees.

On the other hand, the class of toric varieties is a very special one. To cover a wider range of varieties, it pays to study n-dimensional varieties with a lower, namely $k(\leq n)$-dimensional torus action. The goal of the program is to develop a theory parallel to the toric program. However, since there is only a k-dimensional "redundancy" involved, one expects to encode those varieties by a k-dimensional combinatorics and an $(n-k)$-dimensional geometry. Moreover, both sets of data are expected to interact.

One-codimensional torus actions, i.e. $k = n - 1$, have been investigated from different points of view. In one of the early books about toric geometry, Kempf et al. have associated to them a toroidal structure, i.e. analytically locally the situation looked like toric again, and one could associate a fan-like object to the variety. However, there was no one-to-one correspondence between geometry and combinatorics anymore.

Later on, one-codimensional group actions, often referred to as "complexity one" actions, were dealt with for the case of surfaces, [FZ03a], [FSZ07], and in the context of spherical varieties, [Arz97], [Tim97]. The latter means that the torus is replaced by a reductive group, and the complexity measures the codimension of the action of a Borel subgroup. In particular, the original spherical varieties are those of complexity 0 then.

Eventually, the same problem was treated in symplectic mathematics, too. Here, in [KT03], one has even studied the case of complexity ≤ 2.

1.2.3 Group actions and Derived Categories

It is quite common that a group acts on the data one wants to study. Usually, part of the interest goes then to the existence and properties of the quotient. In algebra and algebraic geometry, group actions and their quotients have long been studied, leading to Geometric Invariant Theory which generalises the classical invariant theory from algebra.

From the point of view of derived algebraic geometry, where the derived categories of coherent sheaves of varieties are considered as interesting objects of study, examining group actions is fruitful, as well. In particular, the topic of equivariant equivalences (Fourier-Mukai transforms) has been targeted.

The most basic result on group actions and derived categories is the celebrated derived McKay correspondence by Bridgeland, King, Reid [BKR01]. As it provides a statement on quasi-projective varieties, it is valuable both in the local and the global case.

Concerning equivariant Fourier-Mukai transforms, one case of actions on surfaces (covering actions coming from the canonical bundle) has been studied in [BM98].

1.2.4 Special metrics in low dimensions

We consider Riemannian metrics which are induced by differential forms enjoying special algebraic properties. This approach is particularly fruitful in dimensions less than or equal to 9. Important examples which can be treated in this framework are for instance, G_2–, spin(7)– or Calabi–Yau–manifolds (in dimensions 7, 8 or $2m$ respectively) which play an important rôle in Riemannian geometry. In physics, these geometries naturally appear in connection with flux compactifications of heterotic/type I string– or M–theory. We focus mainly on the investigation of topological and geometrical properties of manifolds with special metrics, construction of (compact) examples, the study of special ("calibrated") submanifolds, and the interplay between different special geometries in low dimensions.

To define Riemannian metrics by means of forms is a viewpoint particularly emphasised in recent work by N. Hitchin. This led, for instance, to *Hitchin's variational principle* [Hit01] whose critical points characterise special geometries in low dimensions. This principle was taken up by Dijkgraaf et al. [DGNV05] in their proposal of topological M–theory and form theories of gravity. A further application of [Hit01] gave rise to the so–called *Hitchin flow* which links weakly integrable Calabi–Yau metrics in dimension 6 to G_2–manifolds. The link between these special metrics in dimension 6 and 7 was more closely investigated in terms of *intrinsic torsion* conditions by Chiossi and Salamon in [CS02]. These materialise as field equations in heterotic/type I string–theory and were successfully used to construct examples of string compactifications on dimension 6 by Cardoso et al. [CCD+03]. Compact examples of G_2– and spin(7)–manifolds were constructed in the seminal work of Joyce [Joy00]. Strongly connected with the form point of view is the notion of a *calibrated submanifold* introduced by Harvey and Lawson [HL82]. They play an important rôle in string– and M–theory, where they account for *branes*. Both for mathematical and physical reasons, the deformation theory of calibrated submanifolds is an important problem, an issue which was settled by McLean for *normal* deformations [Mcl98].

1.2.5 Generalized Albanese Varieties

We consider a smooth projective variety X defined over the complex numbers. There exists a universal cover $\tilde{X} \to X$. Unfortunately, this map is NOT a morphism of algebraic varieties, unless the fundamental group $\pi_1(X)$ is finite. Even worse, the universal cover $\tilde{X}$ is in general not an algebraic variety. All these effects can be observed in the case of algebraic curves. Thus, a description of the universal covering in terms of algebraic geometry seems quite hard.

In topology there exists a CW-complex $K(G, 1)$ which is determined by the group G. For any CW-complex X there exists a map $X \to K(\pi_1(X), 1)$ which induces an isomorphism of fundamental groups. It is this map we want to generalize. Its existence in the case of abelian fundamental groups is given by the classical Albanese morphism.

1.3 Results and their Interpretation

1.3.1 Smoothings and Degenerations of CYs

In [vS06] we have introduced so-called flag quivers. They are characterized by the presence of exactly one source and sink among the vertices. The projective toric varieties associated to the corresponding flow polytopes are then special singular Fano varieties. Our main result is that the affine cones over those varieties are always smoothable in codimension three. Under certain additional, easily checkable conditions on the quiver, those smoothings do also exist in degree 0, i.e. they provide smoothings (in codimension three) of the original projective variety itself. The extra condition asks for the existence of detours avoiding certain vertices in the given quiver. A special case of the smoothings obtained by the above theorem is the toric degenerations of Grassmannians.

Another direction of research was the investigation of Gröbner degenerations of Calabi Yau varieties. In the diploma thesis [Joh07], we have addressed this question for hypersurfaces in projective toric varieties given by (reflexive) polytopes. For a given term order, Sturmfels has shown that the ambient space degenerates into a monomial scheme related to a certain subdivision of the given polytope. The corresponding limit of the hypersurfaces provides a codimension one subcomplex of this subdivision. The result of the thesis was that not all those complexes are obtained that way.

Finally, in [AC04] and [Stu05], we have investigated the deformation theory of the most degenerated, namely the monomial ideals. While in the former paper the infinitesimal deformation and obstruction spaces were calculated, we used Gröbner basis methods for studying the true adjacency relations of these monomial ideals in (multigraded) Hilbert schemes. We have approached the 1-skeleton of these Hilbert schemes by studying the space of so-called edge-ideals connecting two given monomial ideals.

1.3.2 The category of T-Varieties

In [Hau06], we have established an equivalence of categories between affine varieties with an effective T-action, on the one hand, and so-called polyhedral divisors D on the Chow quotient $Y = X/T$ satisfying a certain positivity condition. Turning the latter objects into a category required the definition of morphisms. This could be done in a very natural way which is strongly related to the toric situation. In particular, the face relation among the polyhedral coefficients of the divisors plays an important rôle. As usual, it refers to open embeddings among the T-varieties.

In the paper [AHS06], we make use of the face relation mentioned above and describe arbitrary T-varieties as glueings of affine, open, T-invariant subsets. For this, one has to adapt the notion of a polyhedral divisor: Their polyhedral coefficients are now promoted to polyhedral complexes. An important example for X is the Grassmannian $\mathrm{Grass}(d,k)$ with its $(\mathbb{C}^*)^{k-1}$-action. If $d=2$, then Y becomes the moduli space $\overline{M_{0,k}}$ of k-pointed, stable rational curves, the participating prime divisors are those arising from partitions of the k points, and the polyhedral coefficients are related to the Weyl chambers of the corresponding root system, [AH06].

The case of a complexity one action was investigated in [Vol07]. The result is an easy way to obtain Kempf's fan of the toroidal method from the polyhedral divisor. The description via polyhedral divisors contains some more information – this is essential for being able to recover the T-variety from the combinatorial data.

The description of T-varieties (affine or not) by polyhedral divisors provides another feature. The combinatorial structure of the polyhedral coefficients reflects nicely the T-orbit decomposition of X. In particular, analogously to the case of toric varieties where orbits correspond directly to faces, this makes it possible to observe their adjacency relation and to get hands on the orbit closures. The report [Alt07] covers this material and describes a possible relation of polyhedral divisors to the FM-transform among derived categories.

1.3.3 Group actions and Derived Categories

There is a consistent theory of equivariant Fourier-Mukai transforms in the case of finite group actions [Plo07]. The results have been applied to Hilbert schemes and provide some statements about their birational geometry.

Also, by Balmer's theory [Bal05] on can associate to any tensor triangulated category a ringed space. This has been examined, among other cases, in [Sos07] for finite group actions. The resulting spectrum of the equivariant category is the variety divided by the group action. This is remarkable insofar as the equivariant category is equivalent to the derived category on the smooth stack: $\mathrm{Spec}(D^G(X)) = X/G$ and $D^G(X) \cong D^b([X/G])$.

1.3.4 Special metrics in low dimensions

We found a compact, non–symmetric example of a $PSU(3)$–metric, a problem risen by Hitchin. Further, we showed that this type of geometry is similar in nature to quaternionic Kähler geometry in dimension 8, in as far as both are characterised by the existence of certain *Rarita–Schwinger fields*. This gave a new characterisation of quaternionic Kähler metrics in dimension 8. As a result, we could derive a new integrability condition on their Ricci tensor.

Further, we studied the deformation theory of calibrated submanifolds of a G_2–manifold, whose boundary is constrained to lie in a fixed second calibrated submanifold. This constitutes the first step towards a construction of an analogue for G_2–manifolds of Floer homology in symplectic geometry (joint work in progress with D. Gayet (ICJ Lyon)).

1.3.5 Generalized Albanese Varieties

The main result is Theorem 4.2 in the article [Hei06] which says that a certain determinant line bundle L_r (see the methods section) is nef. Furthermore, it is trivial only on those curves $\iota : C \to X$ with the property that any family E_S of semistable vector bundles on $S \times X$ gives a locally constant family when restricted to $S \times C$.

If we consider the following relation $\preceq$ on nef line bundles by saying $L_1 \preceq L_2$ when $C.L_1 > 0 \implies C.L_2 > 0$ for all curves $\iota : C \to X$, then we obtain $L_r \preceq L_{r+1}$ for all positive integers r. Thus, we obtain a limit line bundle L_∞ with $L_r \preceq L_\infty$ for all r and $L_\infty \preceq L_r$ for $r \gg 0$. Tsuji's nef reduction provides us now with a rational version of the Albanese variety.

1.4 Applied Methods

1.4.1 Smoothings and Degenerations of CYs

In [AvS00] we had calculated several invariants for polytopes. Some of them have an interpretation as graded pieces of infinitesimal deformations (T^1) or obstruction spaces (T^2) of the toric varieties associated to the given polytope. Then, in [vS06] we investigated flow polytopes arising from quivers. For them, the above invariants were better approachable, and we could use the vanishing of certain graded pieces of T^1 and T^2 to decide when the T^2 space is not large enough to obstruct the glueing of local smoothings to a global one.

Usually, one can say something about unobstuctedness only if $T^2 = 0$. However, this was not the case in our class of singularities. In most examples, T^2 does not vanish at all. To obtain our results it was necessary to control the distribution of T^1 and T^2 with respect to the multidegrees induced by the torus action.

1.4.2 The category of T-Varieties

Our project aims at torus actions of arbitrary complexity (this means a generalization since otherwise the complexity was bounded by 2). The description of an n-dimensional variety X admitting a k-dimensional torus action is based on an $(n-k)$-dimensional variety Y carrying the information of X modulo the redundancy provided by the torus action. If X is toric, then Y is a point. In the general situation, Y is the "Chow quotient" X/T. If X is affine, the latter means that Y sits over all GIT quotients with respect to the different linearizations of the trivial line bundle.

As in the toric case, the combinatorial information about X is encoded as polyhedral objects in the lattice $N \cong \mathbb{Z}^k$ that is the dual of the character lattice of the torus. Moreover, there is an interaction between Y and the combinatorial part. The ultimative datum representing the T-variety X is a Cartier divisor D on Y with coefficients being polyhedra in $N \otimes_{\mathbb{Z}} \mathbb{R}$.

1.4.3 Group actions and Derived Categories

Apart from the methods provided by the papers quoted above, standard methods from representation theory and from modern algebraic geometry and homological algebra apply, [Huy06], [**?**].

1.4.4 Special metrics in low dimensions

The main machinery employed to study the geometrical properties of special metrics consists of principal fibre bundle theory and representation theory of compact Lie groups. Further, spin geometric techniques often prove highly useful when investigating the curvature of such metrics, and are crucial to make contact with physics, where spinors naturally arise in supersymmetric models. The investigation of topological obstructions to the existence of such metrics, as well as their topological properties, involves the standard techniques of homotopy theory, topological K–theory and characteristic classes. Moreover, construction of compact examples or deformation theory of submanifolds often makes use of elliptic PDE theory.

1.4.5 Generalized Albanese Varieties

We considered the Albanese variety as the moduli space of line bundles on the Picard torus of X. This way, we obtain the classical Albanese variety and all its properties. As it is natural in algebraic geometry we try to construct functions on $K(\pi_1(X), 1)$ in order to present the space as the spectrum of the ring generated by these functions.

As usual we could consider only a subring (a sequence of subrings) by using representations $\pi_1(X) \to \mathrm{GL}_r$. Let us sketch the construction shortly: We consider a very ample curve $Y_r \subset M_r(X)$ in the moduli space $M_r(X)$ of all semistable rank r bundles on X. Normalizing this curve, we obtain a universal family on $Y_r \times X$. Using the theory of generalized Theta functions on the moduli space of vector bundles on curves, we obtain a determinant line bundle L_r on X.

1.5 Individual Projects

1.5.1 Smoothings and Degenerations of CYs

1.5.2 The category of T-Varieties

1.5.3 Group actions and Derived Categories

1.5.4 Special metrics in low dimensions

1.5.5 Generalized Albanese Varieties

1.6 Relations within the SFB

1.6.1 Special metrics in low dimensions

In cooperation with Simon Chiossi (HU-Berlin, SFB 647, Project A2) and with Anna Fino (Torino), we are investigating possible relations between G_2 - and $PSU(3)$ - structures. Both geometries arise as ciritical points of Hitchin´s variational principle and have connections to hyperkähler geometry in dimension 4.

1.7 Relations to other Research Work

1.7.1 Smoothings and Degenerations of CYs

The relation to Gross' and Siebert's approach to mirror symmetry was already mentioned.

1.7.2 The category of T-Varieties

Flenner and Zaidenberg have developed a similar theory for the special case of $k = 1, n = 2$. Our language is related, but different. Thus, a generalization becomes possible.

1.7.3 Special metrics in low dimensions

Although $PSU(3)$–structures arise in the same way as G_2–structures via Hitchin's variational principle and are akin to quaternionic Kähler structures in dimension 8 as pointed out above, they also seem to fit into more exotic patterns and to relate to $SO(3)$–structures in dimension 5, see for instance [?] or [Nur06].

1.7.4 Generalized Albanese Varieties

The book [Kol95] of Kollár was an inspiration for this work. There a Shafarevich map was constructed. However, one has to exclude countably many proper cycles from X which leaves classical algebraic geometry. However, there exists a map $\mathrm{Sh}(X) \to \mathrm{Alb}_r(X)$ form Kollárs Shafarevich spaces to our generalized Albanese variety.

Eyssidieux considers in [Eys04] the situation for Kähler manifolds and derives results on the holomorphic convexity of covers of X. Kollár remarked that Eyssidieux's result shows that the above constructed morphisms are regular morphisms and not only rational morphisms.

2 Seit 2008

2.1 Summary

While smoothings of toric singularities are important in physics, the category of toric varieties is to small for carrying a good deformation theory. Thus, it was generalized to the concept of T-varieties, and we would like to study their deformation theory as well as their divivisors and vectorbundles. Moreover, we would like to study minimal resolutions of singularities in the framework of derived and triangulated categories. This creates new objects filling the gap of non-existing geometric varieties.

One major part of the present project is the investigation of T-varieties. They are a common generalization of two well known constructions in algebraic geometry – the theory of (generalized) cones over projective varieties describing so-called good $\mathbb{C}^*$-actions and the theory of toric varieties. While the latter translates algebro-geometric facts or problems into discrete, polyhedral concepts, the general theory of T-varieties reduces the dimension of the algebro-geometric part. If the torus action is k-codimensional, then one has to deal with k-dimensional varieties and certain, additional polyhedral objects. This language has been established in both the local and the global case, and it lead to a good understanding of the configuration of the T-orbits. This generalization of toric varieties was necessary because in A3 we were going to study deformations and smoothing of toric varieties (and their subvarieties that occurr in string theory), but this naturally goes beyond the scope of toric varieties: Since versal deformations (base and total space) inherit the group action on the given space, the category of spaces with a fixed group acting on them provides the right language for studying deformations rather than toric varieties coming with their individual tori.

Taking this as a major motivation for studying T-varieties, the first task is to investigate their deformation theory. However, understanding T-varieties as direct generalizations of the well-established theory of toric varieties, one should follow the general "toric program" and try to save as possible into the generalized situation. In the present project, we would like to begin with studying divisors and vectorbundels on T-Varieties. In particular, this is expected to help finding explicit equations of the smoothings of the singular toric varieties we have proven to exist in the previous A3 project.

The project A3 does focus on local, singular phenomena. Having just discussed deformations of singularities, one is also interested in resolving them.

While resolutions do always exist, for physical reasons one needs only those being as small as possible. However, the higher the dimension, the more seldom those "crepant" resolutions do exist. Hence, we try to replace this concept by a simulated resolution in the context of derived and triangulated categories. In case of quotient singularities this is related to the celebrated MacKay correspondence.

A physical interpretation of geometrical singularities can be given within the set-up of F-theory compactifications. The gauge symmetry is then the corresponding A-D-E group, and charged matter is localized along codimensional stratified submanifolds. Therefore the study of the (non canonical) resolution process of singularities for Calabi-Yau fourfolds and the classification of the cases for which a crepant resolution exists should enlarge the reservoir of known Calabi-Yau manifolds, suitable for string and M/F-theory compactifications. The study should also lead to new insides into F-theory/heterotic duality, which suggests a correspondence between gauge and gravitational anomalies and the Euler characteristic of the elliptic Calabi-Yau manifolds.

2.2 Current Knowledge

2.2.1 Divisors and Vectorbundels on T-Varieties

In [Hau06] and [AHS06], we have developed the affine and global theory of T-varieties, respectively. The idea is to encode the redundancy of a k-dimensional torus (T-) action on an n-dimensional variety X in a k-dimensional combinatorics, i.e. in polyhedral objects contained in the character group of the torus or rather in its dual $N \cong \mathbb{Z}^k$. The remaining information is reflected in an $(n-k)$-dimensional, quasi-projective variety Y (the Chow quotient of X by T) and, for affine X, a polyhedral divisor on Y. The latter means a divisor on Y whose coefficients are polyhedra in $N \otimes \mathbb{R}$, i.e. are exactly the combinatorial objects mentioned before. If X is not affine anymore, then the polyhedral coefficients have to be replaced by polyhedral subdivisions of $N \otimes \mathbb{R}$ – each cell reflecting an affine chart in X.

The special case $n = 2, k = 1$ was treated, in a slightly different language, by Flenner and Zaidenberg in [FZ03b] and [FSZ07]. In the case of an affine surface X, they distinguished between an elliptic, parabolic, and a hyperbolic $\mathbb{C}^*$-action. In the terminology of polyhedral divisors, this corresponds to Y being a projective or an affine curve and to the tail cone being equal to $\mathbb{R}_{\geq 0}$ or $\{0\}$, respectively.

The special case of $n = k$ means that X is a toric variety. Toric Geometry means to deal with X by translating everything into combinatorics – and, indeed, in the above language, the case $n = k$ means that Y is a point. This allows no divisors on Y, and the information of its combinatorial coefficients collapses to the asymptotic behavior of the polytopes or the subdivision, namely to their tail cones or fans, respectively.

For the cases $n = 2, k = 1$ and $n = k$, there exist descriptions of PicX, equivariant vector bundles and their cohomology. In the toric case, these are classical results. The case of $\mathbb{C}^*$-actions on surfaces was treated by Flenner and Zaidenberg.

2.2.2 Deformation Theory of T-Varieties and Equations of Generalized Grassmannians

The deformation theory of (smooth) complex manifolds may be understood as varying the complex structure of the underlying real manifold. The deformation theory of singular spaces on the other hand is useful to study smoothings or at least "improvements" of the often quite unpleasant singularities. In both cases, the general philosophy is that there should be a semi-universal deformation over some base space that includes all possible directions of deformation, which provides important invariants. In the case of complex manifolds or of isolated singularities, the base and total space of the deformation turn out to be decent finite-dimensional complex spaces or schemes. Nevertheless, their structure can be complicated, e.g. singular, reducible, or even non reduced.

The notion of T-varieties seems to provide the right framework for a natural understanding of the already existing deformation theory of toric varieties. To deform an affine toric variety associated with a polyhedral cone $\sigma \subseteq N \otimes \mathbb{R}$, one has to choose a degree $R \in M$ and study the Minkowski splittings of the cross cut $\sigma \cap R^{-1}(1)$.

In the case of a T-variety, the space T^1 of infinitesimal deformations as well as the space T^2 of obstructions inherit a multigrading by the character group M of the torus T. In the case of a good (elliptic) $\mathbb{C}^*$-action on a surface ($k = 1$, $n = 2$), Jonathan Wahl [Wah76] has obtained a description of these vector spaces as cohomology groups of certain sheaves on the quotient T. Similarly, combinatorical descriptions for T^1 and T^2 do already exist in the toric case, i.e. $n = k$.

2.2.3 Matter and the Geometry of Calabi-Yau Fourfolds

Presently there are 4 different ways to obtain four-dimensional theories with $N = 1$ supersymmetry from string/M/F-theory: compactification of the $E_8 \times E_8$ heterotic string, M-theory on G_2 manifolds, F-theory on Calabi-Yau (CY) fourfolds and various intersecting D-brane models. In the course of study of these theories various dualities between them have been obtained. For instance, M-theory on G_2 is expected to be dual to the heterotic string compactified on a T^3 fibered CY-space. Another example is the heterotic string/F-theory duality which leads to a correspondence between elliptically fibered CY n-folds X together with stable vector bundles V (with structure group G contained in $E_8 \times E_8$) and elliptically fibered CY (n+1)-folds Y, which has to admit a section H of ADE singularities. On the heterotic string side H corresponds to the observed gauge group (i.e., the centralizer of G in E_8) under which matter is charged. In other words the chiral matter content of the physical theory is reflected in the singularity structure of Y. To establish the correspondence it is assumed that the elliptic fibrations admit a section σ which does not meet the critical points of the fibers. The case $n = 1, 2$ has been established by D. Morrison and C. Vafa [Vaf96],[MV96a],[MV96b] as well as S. Katz and C. Vafa. The investigations for $n = 3$ started with the work of R. Friedman, J. Morgan und E. Witten [FMW97] and have been continued in [BJPS97],[AC98] [AC99]; as a result, the comparison of the respective moduli spaces gives a relation between the Hodge-numbers of X and Y and first order deformations of V. A comparison of the physical anomalies led to a relation between the Euler characteristic of Y and the secondary Chern-classes of X and V.

To every elliptically fibered CY three- or fourfold (with section) one can associate a Lie group G with representation ρ of G. The group is determined from the Weierstrass model which has singularities that are generically rational double points. These double points lead to local factors of G and are given by either the corresponding ADE groups or some associated non-simply laced groups. Further, ρ is a sum of representations coming from the local factors of G and of other representations which can be associated to curves and points in the discriminant divisor at which the singularities are worse than generic. The vanishing of all anomalies in the physical theory constraints the geometry of possible CY three- or fourfolds and leads to a surprising relation between the CY spaces and the representations ρ. In particular, this gives an explicit formula for the Euler characteristics of the CY spaces in terms of ρ. A proof for CY threefolds has been given by A. Grassi und D. R. Morrison [GM00]. In joint work with G. Curio [AC99] a new method for computing Euler characteristics of CY fourfolds was developed. Moreover, it was possible to give a geometrical proof of a formula for Euler characteristics of certain CY fourfolds which A. Klemm, B. Lian, S.-S. Roan and S.-T. Yau derived in [KLRY98] using toric geometry. A further result of [AC99] is a method which allows to compute the Euler characteristics of (singular) surfaces which are embedded into compact complex manifolds of dimension three

2.2.4 Triangulated Resolutions of Singularities

If X is a germ of an normal surface singularity, then there exists a unique minimal resolution $\pi : \widetilde{X} \to X$; every other resolution factors via π. Besides the fact that uniqness is always nice, this means that $\widetilde{X}$ combines the advantage of being a smooth object, on the one hand, with that of being still close enough to the original X, on the other. In particular, information being read from $\widetilde{X}$ is not much "contaminated" by artificial content arising from unneccessary further modifications.

In higher dimension, this nice and convenient behaviour fails. If the singularity X is a canonical Gorenstein singularity, then crepant resolutions is the right notion replacing minimality at least partially. While crepant resolutions are not unique (e.g. $X = V(xy - zw) \subseteq \mathbb{C}^4$), they do still the job of being a resolution avoiding unneccessarily big exceptional sets. For instance, they preserve the Calabi-Yau property, and they provide the "right" Hodge numbers. However, besides their non-uniqueness, the existence of crepant resolutions is not granted and rather seldom in dimension ≥ 4.

2.2.5 The Degree Stratification on the Toric Hilbert Scheme

Arising from an idea of Arnold, in the 1990s different authors (Peeva, Stillman, Sturmfels, et al) studied the so-called "Hilbertscheme". This scheme parameterises ideals with a fixed multigraded Hilbertfunction, which equals the Hilbertfunction of some ideal I of a given point configuration $\mathcal{A}$. This parameter space first of all contains all specialisations of I obtained by Gröbner degeneration. These correspond to a choice of term orders on $\mathcal{A}$ and are closely related to the combinatorial structure of the Secondary Polytope of the configuration $\mathcal{A}$. In particular, these specialisations form a so-called 'coherent' component of the toric Hilbertscheme. As are all further components, this one is toric itself and is described by the already mentioned Secondary Polytope.

In the meantime research done by Alexeev has picked out the other components which do not contain the original ideal I as a central theme. They are given by generalised Secondary Polytopes. But the different components possess curious intersection behaviour. In general it does not have the form of a polyhedral complex. By now research by Santos has shown that the toric Hilbertscheme need not be connected; like other base spaces of deformations it is not necessarily reduced. Using a flip construction it is possible to "run" along the edge ideals through the Hilbertscheme – an effective computer program for this has been written ("Tigers" by MacLagan and Thomas).

2.3 Expected Results and their Intepretation

2.3.1 Divisors and Vectorbundels on T-Varieties

Since the theory of toric varieties is a natural special case of the theory of T-varieties or polyhedral divisors, there is the general plan to follow the toric program and generalize it to the case of lower-dimensional torus actions.

Since polyhedral divisors have been developed along the lines of toric varieties, one should continue to follow the whole toric program. In particular, one should understand divisors, rational equivalence, the Chow ring, sheaves and their cohomology, singularities and their resolutions and deformations. Since the above method includes also non-affine T-varieties, it should provide a useful tool to construct equivariant compactifications. Moreover, one could try to understand equivariant vector bundles and their Chern classes. A good knowledge of these would lead to an understanding of the moduli space of coherent sheaves via the localization method.

The description of non-affine T-varieties required the choice of an affine, open, T-invariant covering – and the result does truely depend on it. Is there any weakening of the data that corresponds to a forgetting of this choice?

One should establish the relation to Fourier-Mukai transforms. If this became true, then the whole method would apply to other classes of varieties going beyond torus actions.

Eventually, T-varieties might become important in coding theory. There are already approaches using toric varieties – and our method (for complexity one actions) could be used to mix them with codes coming from curves.

In particular, in this part we are aiming for an understanding of equivariant Weil- and Cartier divisors on X (to be distinguished from the polyhedral divisor on Y which is part of the datum determining X), to a description of $\operatorname{Pic} X$, and, eventually, to a theory of equivariant vector bundles or coherent sheaves on T-varieties.

2.3.2 Deformation Theory of T-Varieties and Equations of Generalized Grassmannians

For deformations of toric varieties, the torus T acts on the total space of the versal deformation, which hence allows a description in terms of polyhedral divisors. In particular, this language provides a more natural framework for studying deformations. So, a first task is to transfer existing results about toric deformation theory into the language of polyhedral divisors. Furthermore, we want to understand how to combine different one-parameter deformations, in particular in the case of non-negative degrees. This should allow us to describe the components of the versal deformation as T-varieties.

Furthermore, we would like to use the multigrading by the character group M of the torus T to understand the infinitesimal deformation space T^1 as well as the space T^2 containing the obstructions for deforming X. These vector spaces or their M-homogeneous pieces should be described as cohomology groups of certain sheaves on Y.

In their work on the minimal model program, Gavin Brown and Miles Reid have found a class of four parameter smoothings of certain reducible, singular surfaces consisting of toric components [Bro06]. The total space is six-dimensional and carries a four-dimensional torus action. Understanding these varieties via polyhedral divisors will provide examples of interesting deformations of toric varieties and allow us to approach the problem of combining one-parameter deformations of toric varieties. More importantly, this example suggests that T-varieties with the language of polyhedral divisors might prove useful in even more applications, including the minimal model program.

Another aim of our research of deformations of toric varieties is to obtain explicit description of smoothings where we only have existence results so far. In [vS06], we have proven that certain singular toric varieties provided by quivers are smoothable in codimension three, cf. Report A3-**??**. An interesting special case is that of a toric degeneration of the Grassmannian to a quiver variety, which is also observable with the methods of varying polyhedral divisors mentioned above. Since the Grassmannian has a nice, explicit description by the Plücker relations, we would like to find similar equations for the general smoothings of toric quiver varieties.

2.3.3 Matter and the Geometry of Calabi-Yau Fourfolds

In case of elliptic surfaces with a section the types of possible degenerated fibers (corresponding to the extended Dynkin-diagrams of type A-D-E) have been classified by Kodaira. The goal of this project is to study the (non canonical) resolution process for CY fourfolds and to classify the cases for which a crepant resolution exists, thereby enlarging the reservoir of known CY fourfolds suitable for string and M/F-theory compactifications. Furthermore, an expected relation between certain group representations and Euler characteristics of CY fourfolds should be studied.

The aim of this project is to study elliptically fibered CY fourfolds Y of the following structure: the base B of Y is assumed to be a $\mathbb{P}^1$ bundle $B \to B_2$ with section R where B_2 is given by a rational surface (for instance the Hirzebruch-surface or del Pezzo surface). Further it is assumed that the elliptic fibration $\pi\colon Y \to B$ has a section which does not intersect with the critical points of the fibration. Now if one assumes that all fibers of Y are irreducible then Y can be shown to be isomorphic to its Weierstrass model $\pi_o\colon Y_0 \to B$ (together with a morphism $F\colon Y \to Y_0$ with $\pi_0 \circ F = \pi$). The Weierstrass model is determined by a line bundle L on B and by two sections g_2, g_3 of $L^{\otimes 4}$ and $L^{\otimes 6}$ satisfying $4g_2^3+27g_3^2 \not\equiv 0$. CY fourfolds of this kind have been studied for instance in [FMW97], [DGW96], [Gra97], [AC99], [AC98].

If one allows degenerate fibers then the question occurs when does a crepant resolution of singularities of the Weierstrass model exist, that is, resolutions which preserve the canonical class of Y. This question will be studied in this project, in particular, sufficient conditions for crepant resolutions should be derived. Degenerate fibers of elliptic surfaces with one section have been classified by Kodaira; the irreducible components of the fibers are smooth rational curves with transversal intersection and whose dual graph corresponds to an extended Dynkin-diagram of type ADE. In case of CY fourfolds one expects that the dual graph corresponds to new fibers which did not occur in Kodaira's list. For instance, if R is a component of $G_2 = \mathrm{div}(g_2)$,

$G_3 = \mathrm{div}(g_3)$, $D = \mathrm{div}(4g_2^3 + 27g_3^2)$ and if $G_2 = R + S$, $G_3 = aR$ and S, R have transversal intersection, then a crepant resolution exist for a even. The dual graph corresponding to the fiber over 0 is given by $\overset{1}{\bullet} - \overset{2}{\bullet} - \overset{1}{\bullet}$ and does not occur in the surface case.

After these investigations it is planned to compute the topological invariants (Chern classes and Euler characteristics) of the spaces with an eye on physical applications. The computation of the Euler characteristics can essentially follow the lines of [AC99] using a stratification method. This method uses the structure of the fibers and that the Euler-characteristic behaves well under addition and multiplication; the Euler characteristics $\chi(F_i)$ will be computed for every fiber type which occurs over the corresponding stratum D_i. $\chi(D_i)$ can be computed using the so-called "Plücker formulae" for surfaces derived in [AC99].

The investigations in this project will enlarge the reservoir of CY fourfolds and will lead to new models for string theory. The computation of the Euler characteristics and Hodge numbers of the spaces will give mathematical evidence for the relations expected from physical duality. Moreover, a classification of divisors with arithmetic genus equal to 1 will give new examples of non-perturbative superpotentials in M/F-theory compactifications.

2.3.4 Triangulated Resolutions of Singularities

It is an idea of Bondal to focus on the derived category $\mathcal{D}(\mathrm{Coh}\widetilde{X})$ of coherent sheaves on a resolution $\widetilde{X}$ rather than on the true geometric resolution $\widetilde{X}$ itself. If one has a good notion of smoothness of a triangulated category, then this broadens the concept of resolutions to a categorial point of view. This might have two effects: First, certain new "triangulated resolutions" might occur, i.e. given by smooth triangulated categories being not derived categories of a geometric situation. Second, certain different geometric (e.g. small, crepant) resolution might induce the same derived category, i.e. as triangulated resolutions they are becoming identified, cf. [Bri02]. In particular, Bondal's conjecture is that minimal triangulated resolutions do always exist.

We would like to investigate the existence of unique, minimal resolutions in the context of triangulated categories or, as a first step, in the context of K-theory.

Eventually, we would also like to study the situation of normal surface singularities. Here, as said in the beginning, the existence of minimal resolutions is no problem – but here we are able to study their behavior under deformations, e.g. on the Artin component. Every information we can exploit out of the derived categories of the minimal resolutions of the fibers might be a fact that generalizes to the minimal triangulated resolutions and would show their meaning.

2.3.5 The Degree Stratification on the Toric Hilbert Scheme

The project at hand should at first apprehend the curious intersection behaviour of the components of the Toric-Hilbert Scheme in the language of the Secondary Polytopes. In this context it would be interesting and maybe helpful to examine the stratification given by the maximal grading. On the coherent component this reflects the exact orbit stratification; on the other components I expect more detailed information. Three special cases seem to be clear: If the toric grading is maximal then the corresponding ideal has to be an "inner point" of the coherent component. In the other extreme case, if the maximal grading is given by $\mathbb{Z}^n$ (with $n = \#\mathcal{A}$) then it is a monomial ideal. These are the most special points of the Hilbertscheme and they are spread over all components. Lastly we also understand the $n-1$ dimensional gradings. These correspond to the edge ideals, which form a frame inside the Hilbertscheme, see [Stu05].

A local description of the toric Hilbertscheme was coined by Peeva and Stillman – in a neighbourhood of monomial ideals. This description should be compared to the base space of the versal deformation. For squarefree monomial ideals formulas for T^1 and T^2 (describing the infinitesimal theory) have already been found, see [AC04]. Especially results in the isolated points of the Hilbertscheme constructed by Santos would be interesting. Corresponding to that, does $T^1 = 0$ hold as well?

2.4 Methods

2.4.1 Divisors and Vectorbundels on T-Varieties

The theory of polyhedral divisors is a useful tool to study equivariant compactifications of T-varieties. In particular, the comparison of line and vector bundles on X and on those compactifications $\overline{X}$ seems to be interesting. Moreover, these methods look promising for an investigation of framed bundles on $X \subseteq \overline{X}$. A goal on the horizon could be the usage of the localization theorem to obtain information about the moduli space of all (framed) bundles.

The equivarint ones form the fixed point set within this space. Maybe, the understanding of these moduli spaces goes so far to produce strong invariants of X distinguishing it from different complex structures.

In the special case of X being the Russell cubic, the understanding of the moduli space of framed bundles was a (not finished) project of Kurke/Lehn/Teschke to provide another approach to its exotic $\mathbb{C}^3$ structure. We hope that our concept might help here. The Russell cubic admits a $\mathbb{C}^*$-action, with Y becoming the blow up of $\mathbb{C}^2$ and the tail cone being $\{0\}$.

2.4.2 Deformation Theory of T-Varieties and Equations of Generalized Grassmannians

While the notion of splitting polytopes into Minkowski summands showed up somewhat surprisingly in the language of, for instance, [Alt97], it shows up naturally in the context of T-varieties: If X is given by a polyhedral divisor $\mathcal{D}$ on $Y = \mathbb{P}1$, and if $\mathcal{D}$ equals the sum of the prime divisors $\{0\}$, $\{1\}$, $\{\infty\}$ with polyhedral coefficients Δ_0, Δ_1, Δ_∞, respectively, then moving the prime divisors on $\mathbb{P}1$ as $\mathcal{D}_t := \Delta_0 \otimes \{0\} + \Delta_1 \otimes \{t\} + \Delta_\infty \otimes \{\infty\}$ creates a flat family X_t with the non-toric X as fiber over the starting configuration $t = 1$. For $t \to 0$, i.e., when two of the prime divisors coincide, the corresponding coefficients are replaced with their Minkowski sum. The special fiber X_0 described by $\mathcal{D}_0 = (\Delta_0 + \Delta_1) \otimes \{0\} + \Delta_\infty \otimes \{\infty\}$ is a "toric configuration" on $\mathbb{P}1$, that is, the torus action can be upgraded to turn X_0 into a toric variety. Polyhedral divisors also help in understanding one-parameter deformations in non-negative degrees: Here, the total space is not toric, however, it does admit an action of the torus that acts on the special fiber. This results in a polyhedral divisor on $\mathbb{A}1 \times \mathbb{P}1$ with a non-toric configuration of prime divisors.

2.4.3 Triangulated Resolutions of Singularities

The method used by now is to look for minimal triangulated resolutions of a singularities among those triangulated categories having a certain meaning, i.e. arising quite naturally from X. For instance, if X is a quotient singularity, then one might enforce the McKay-correspondence of [BKR01] by considering the equivariant derived category on the smooth space where the group is acting on. Another possibility is to enforce tilting theory by looking for a triangulated category among those describing modules over non-commutative, finite-dimensional algebras, cf. [SvdB06]. In the case of a rational singularity

X, Kuznetsov used in [Kuz06] special semiorthogonal decompositions of the derived category $\mathcal{D}(\mathrm{Coh}E)$ if E is the exceptional set of some (geometric) resolution. Eventually, in [Kal06] there is also a symplectic approach to this problem. This indicates a possible collaboration with symplectic projects of the SFB.

Our approach to construct minimal triangulated resolutions is different. Starting with a singularity X and two true geometric resolutions $\pi_i : \widetilde{X}_i \to X$ $(i = 1, 2)$, we would like to construct a (not minimal yet) triangulated resolution $\mathcal{T}$ that lies under both $\mathcal{T}_1 = \mathcal{D}(\mathrm{Coh}\widetilde{X}_1)$ and $\mathcal{T}_2 = \mathcal{D}(\mathrm{Coh}\widetilde{X}_2)$. Then, we need a generalization of this method to construct such a $\mathcal{T}$ even for non-geometric $\mathcal{T}_1$ and $\mathcal{T}_2$. Eventually, some numerical invariant is needed to ensure that the whole process will stop after finitiely many steps. A first approach to this plan is the investigation of e.g. four-dimensional, toric, canonical Gorenstein singularities that do not allow crepant resolutions. Here one can consider pairs of toric resolutions and try to construct a $\mathcal{T}$ as mentioned before.

2.4.4 The Degree Stratification on the Toric Hilbert Scheme

A useful approach would be to study the versal base space of monomial ideals in such settings, where this one is already known – e.g. for the Stanley-Reisner-ideal of the simplicial partition of S^3 with 7 vertices. Is it possible to see the stratification of the grading?

In algebraic geometry deformationtheory is understood functorial. Are there subfunctors which might collect rather easy deformations, e.g. exactly the Gröbner degenerations? One approach has recently been made by Olsson from Berkeley to, for example, realise strictly the coherent component of the toric Hilbertscheme functorial. How does then or in Olsson‘s example the tangentspace look like?

Finally there is yet a link to the topics "Divisors and Vectorbundels on T-Varieties and "Deformation Theory of T-Varieties and Equations of Generalized Grassmannians". In the last years an encoding of affine varieties with torus action by using polyhedral divisors on lower dimensional varieties has been constructed in [Hau06] . Concretely, these are the usual Cartier divisors – but one also allows convex polyeder with Minkowskisum structure as coefficients. Fibrations arise within as intermediate steps, which have toric varieties as fibers (or, in some special points, are unions of those). The coherent component seems to be an universal object for such a fibration –

this means our construction should induce a mapping into this component. In this context it would be important to know why the other components do not appear. Or conversely, what happens when I start with a family contained in a different component – do I get at least a construction similar to a polyhedral divisor? For this the known characterisations of morphisms into toric varieties should be used (Cox; A'Campo/Hausen/Schröer).

2.5 Projects

2.5.1 Divisors and Vectorbundels on T-Varieties

2.5.2 Deformation Theory of T-Varieties and Equations of Generalized Grassmannians

2.5.3 Matter and the Geometry of Calabi-Yau Fourfolds

2.5.4 Triangulated Resolutions of Singularities

2.5.5 The Degree Stratification on the Toric Hilbert Scheme

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