1 [B3] Singularity Structure, Longtime Behaviour, and Dynamics of Solutions of Nonlinear Evolution Equations

1.1 Summary

Many central problems in geometry, topology and mathematical physics reduce to questions regarding the behavior of solutions of nonlinear evolution equations. Examples are Hamilton's classification of compact 3-manifolds with positive Ricci curvature and the work of Christodoulou and Klainerman on the nonlinear stability of Minkowski spaces. Furthermore the Ricci flow seems to be a promising tool in settling the Poincaré conjecture.

The Ricci flow deforms the metric of a Riemannian manifolds along their Ricci curvature and yields parabolic equations. Semilinear reaction-diffusion equations are also of parabolic type. Examples of hyperbolic equations include the Einstein field equations of general relativity as well as semilinear wave equations.

In all these equations, the global dynamical behavior of bounded solutions for large times is of significant interest. Specific questions concern the convergence to equilibria, the existence of periodic, homoclinic, and heteroclinic solutions, and the existence and geometric structure of global attractors. On the other hand, many solutions develop singularities in finite time. The singularities have to be analyzed in detail before attempting to extend solutions beyond their singularities, or to understand their geometry in conjunction with globally bounded solutions. In this context we are particularly interested in global qualitative descriptions of blow-up profiles.

It is the aim of this project to study the singularity formation and the long-term dynamical behavior of solutions of specific nonlinear evolution equations. In particular we hope to combine different approaches, i.e. to actively search for techniques which were successful for one type of equation and to adapt them to other equations. Relevant methods involve elliptic, parabolic and hyperbolic PDE theory, differential geometry, general relativity, nonlinear analysis, dynamical systems, spectral theory, calculus of variations, and geometric measure theory.