1 [B4] Geometry and Physics of Spacelike Hypersurfaces in Lorentzian Manifolds

1.1 Summary

Major aim of the project is the understanding of stationary and evolutionary spacelike hypersurfaces of prescribed curvature in Lorentzian manifolds with the help of new analytical and geometric techniques in nonlinear partial differential equations. One aspect here is the use of better a priori estimates for the slope and the curvature of such surfaces to control their properties and to establish existence of natural time-gauges. For example, new estimates for the slope of constant mean curvature surfaces can be used to establish the existence of asymptotically hyperbolic slices in a wide class of models for isolated gravitating systems, allowing a more detailed investigation of gravitational waves in the exterior region. Similarly, prescribed mean curvature gauges are used to study the stability behavior of cosmological models both near singularities and their longtime behavior.

In addition to studying suitable embeddings of spacelike slices in Lorentzian ambient manifolds, an important part of the project is the understanding of the Cauchy data on each individual slice. Each slice carries information on mass-momentum density of the system modelled. It is then a crucial task to find invariant geometric structures of the slice representing such physical concepts as mass, energy, or momentum of the whole system or parts of it. In the last two decades considerable progress was made in understanding the concept of total mass and energy, with some understanding also of the total linear momentum. This has provided new geometric and analytical tools for aiming at successful invariant concepts also for total angular momentum as well as quasi-local definitions of these concepts. In particular, the theory of hypersurfaces satisfying suitable variational problems or evolution equations will prove to be a very strong method in the description of such physical quantities. A longterm goal is a complete set of global and quasi-local geometrically invariant structures for all preserved physical quantities with a precise and consistent description of their interrelation and their interaction under the Einstein evolution equations.

Concerning energy inequalities on 3-dimensional slices of isolated gravitating systems,it is planned to investigate the extension of energy inequalities such as the Penrose inequality from timesymmetric and maximal slices to

more general Cauchy data and to quasi-local settings. This will involve more general foliations of 3-manifolds by two-dimensional spheres satisfying variational principles involving general Cauchy data rather than just the 3-metric of the slice. While up to now mostly variational properties involving area have been exploited, recent progress on the Willmore functional of 2-surfaces suggests that it will be fruitful to consider curvature functionals in this context.