[A11] Algebraic Varieties and Principal Bundles: Semistable Objects and their Moduli Spaces

Project A11 focusses on two central themes in algebraic geometry: The classification of algebraic structures (e.g. projective manifolds, principal bundles over a fixed projective manifold) by moduli spaces and the investigation of special manifolds. Detailed aspects of the geometry of certain moduli spaces will be investigated (Farkas, Schmitt) as well as general aspects of the construction of moduli spaces (Schmitt). The objects and methods of this project closely relate to mathematical physics:

1) Any representation $\rho: G \longrightarrow \mathrm{GL}(V)$ of a reductive Lie group leads to a gauge theoretic moduli problem on a fixed projective manifold X. The solution to this moduli problem may be obtained by gauge theory as a real analytic space or by geometric invariant theory as a quasi-projective scheme. According to the universal Kobayashi-Hitchin correspondence, both approaches yield the same real analytic space. The resulting moduli space also maps projectively to the GIT quotient V//G of Hilbert and Mumford which is called the "moduli space of vacua". These moduli problems are physically most interesting, if V//G satisfies the Calabi-Yau condition. Examples of suitable representations were discovered by Hanany and Vegh via dimers and tilings. These are examples of so-called quiver gauge theories ("AdS/CFT dual quiver gauge theories for D3-branes"). Moduli spaces of quiver representations appear explicitly in our project.

2) Principal bundles and connections are fundamental tools for the description of nature through mathematics. Classical work of Donaldson, Ramanathan, Subramanian and others shows that the moduli space of Einstein-Hermitian connections (i.e., the minima of the Yang-Mills functional) on a given differentiable principal bundle on a projective algebraic manifold X carries the structure of a quasi-projective scheme, namely the moduli space of stable principal bundles on X. In our project, we propose to study these moduli spaces which origin from relativity theory with the tools of Algebraic Geometry.